Izmir Institute of Technology

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ME 312 Machine Elements II

Project Report

Group 10

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1 INTRODUCTION

This project covers the mechanical design of a three-speed gearbox to be used between the rotor shaft of an H-Darrieus vertical-axis wind turbine and an electrical generator. The aerodynamic design and performance analyses of the wind turbine were carried out by the ESE312 Energy Systems Engineering group, and the rotor speed, power, and torque values under different wind conditions were obtained from these analyses. Within the scope of the ME312 Machine Elements II project, these data were used to determine the required transmission ratios, establish the gear geometry, and evaluate the gears against bending and contact fatigue. The wind turbine rotor operates at different rotational speeds depending on the wind conditions, whereas the generator is intended to operate at approximately 1500 rpm. Therefore, a multi-ratio gearbox is required to increase the rotor speed and bring the generator close to its target operating speed over a wide range of operating conditions. The proposed gearbox consists of three shafts: an input shaft, an intermediate shaft, and an output shaft. The transmission system is divided into two main stages, referred to as Stage A and Stage B. Stage A provides a constant speed increase between the input and intermediate shafts, while Stage B provides three selectable transmission ratios, designated as V1, V2, and V3, between the intermediate and output shafts. Spur gears were selected for both Stage A and Stage B. Although spur gears generally produce more noise than helical gears, gearbox noise was not specified as a design limitation in the project requirements. Therefore, spur gears were preferred because of their simpler geometry, lower manufacturing cost, easier assembly, and absence of axial gear forces. The mechanical power produced by the wind turbine rotor is first transmitted to the input shaft. In Stage A, the spur gear mounted on the input shaft drives the spur pinion mounted on the intermediate shaft, thereby increasing the rotational speed. The resulting motion is then transferred through Stage B. Three different spur gears corresponding to V1, V2, and V3 are located on the intermediate shaft. Depending on the rotor operating condition, the appropriate gear pair is selected. The selected Stage B gear drives its corresponding pinion on the output shaft, and the output shaft subsequently transmits the increased rotational speed to the generator. Since all three Stage B gear-pinion pairs operate between the same intermediate shaft and the same output shaft, their center distances must be equal. Therefore, the three Stage B gear sets were designed to satisfy an identical center-distance requirement. In this way, Stage A provides a fixed speed increase, while Stage B adapts the total transmission ratio to different wind and rotor-speed conditions. The gear design was performed according to AGMA procedures. The gears were evaluated against both bending fatigue and surface contact fatigue. Standard full-depth spur gears were considered, and the main design criteria included standard module selection, prevention of undercutting, face-width requirements, service life, reliability, compactness, and the overall factor of safety.

1.1 Design Inputs Obtained from the Energy Systems Group

The main mechanical design inputs were obtained from the aerodynamic analyses performed by the ESE312 group. The turbine was designed as a three-bladed H-Darrieus vertical-axis wind turbine with a rotor radius of 1.0 m and a blade height of 1.5 m. Three representative operating points were considered to determine the required gearbox transmission ratios. These operating points represent low, intermediate, and maximum wind conditions. The corresponding rotor speed, power, and torque values supplied by the Energy Systems group are presented in Table 1.

Table 1. Wind-Turbine Operating Conditions Provided by the Energy Systems Group

Operating Condition	Wind Speed (V)	Rotor Speed (n)	Angular Speed (w)	Rotor Power	Rotor Torque
Low Condition	3 m/s	93 rpm	9.74 rad/s	33.6 W	3.45 Nm
Intermediate Condition	7 m/s	217 rpm	22.72 rad/s	427 W	18.79 Nm
Maximum Condition	15 m/s	350 rpm	36.65 rad/s	3730 W	101.8 Nm

The maximum design condition was used as the basis for Stage A strength calculations because it produces the highest transmitted power and rotor torque. Accordingly, the bending and contact stress analyses of Stage A were based on a power of 3730 W, a rotor speed of 350 rpm, and a rotor torque of approximately 101.8 N·m. The target generator speed was taken as approximately 1500 rpm. The transmission ratios required for the different rotor operating conditions were determined from the relationship between rotor speed and generator speed. The allocation of these ratios between Stage A and Stage B, the selection of the individual gear pairs, and the resulting gear geometries are presented and evaluated in the following sections.

2 STAGE A DESIGN

Stage A is the fixed-ratio first stage of the gearbox and transmits mechanical power from the input shaft to the intermediate shaft. The wind-turbine rotor is connected to the input shaft, where the larger spur gear acts as the driving member. This gear drives the smaller spur pinion mounted on the intermediate shaft. Since a larger gear drives a smaller pinion, Stage A operates as a speed-increasing gear pair.

The maximum operating condition supplied by the Energy Systems group was selected as the design condition for Stage A because it produces the highest transmitted power and rotor torque.

Table 2. Stage A Design Inputs and AGMA Parameters

Input Power	3730 W
Input Rotational Speed	350 rpm
Selected Number of Driving Gear on the Input Shaft	55T
Selected Number of Driven Pinion on the Intermediate Shaft	16T
Standard Full Depth	k = 1
Pressure Angle	20 degree
Helix Angle	0 degree
Selected Standard Module	3.5 mm
Selected Face Width	51 mm
Grade 2 Through-Hardened Steel	HB = 350
The Required Service Life	80000 hours
Overload Factor	$K_o = 1.25$
Dynamic Factor	Calculated in Report
Size Factor	$K_s = 1$
Hardness Ratio Factor	$Z_W = 1$
Load Distribution Factor	$K_H = 1$
Rim Thickness Factor	$K_B = 1$
Temperature Factor	$Y_\theta = 1$
Reliability Factor	$Y_Z = 1$

The Elastic Coefficient	$Z_E = 191\sqrt{MPa}$
Quality Number	$Q_v = 7$
Surface Condition Factor	$Z_R = 1$

2.1 Input Torque and Transmission Ratio

The input torque transmitted by the wind-turbine rotor is calculated from the relationship between power and angular velocity.

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(350)}{60} = 36.65\ \frac{rad}{s}$$

The transmitted torque is then calculated as:

$$T = \frac{P}{\omega} = \frac{3730\ W}{36.65\ \frac{rad}{s}} = 101.77\ Nm$$

The transmission ratio of Stage A is calculated from the number of teeth on the driving gear and the driven pinion:

$$i_a = \frac{N_g}{N_p} = \frac{55}{16} = 3.44$$

The intermediate shaft speed ,for all condition, is calculated as:

$$n_{int} = n_{in} * i_a = (350,217,93) * (3.44) = (1203.13, 745.94, 319.69)\ rpm$$

2.2 Undercut Control

Undercutting occurs when the cutting tool removes material from the tooth root during manufacturing. This reduces the tooth-root thickness and weakens the tooth. Since the pinion has fewer teeth than the gear, the undercut check is performed for the pinion.

For a standard full-depth spur gear pair, the minimum allowable number of pinion teeth is calculated from:

$$N_{p,\min} = \frac{2k}{(1+2i_a)\sin^2\phi}\left[i_a + \sqrt{i_a^2 + (1+2i_a)\sin^2\phi}\right]$$

All values are substituted:

$$\frac{2(1)}{(1+2(3.44))\sin^2 20°}\left[3.44 + \sqrt{3.44^2 + (1+2(3.44))\sin^2 20°}\right] = 15.21$$

$$N_p = 16 > 15.21 \; No \; Undercut$$

2.3 Gear Geometry and Face Width Selection

Since Stage A uses spur gears, the normal and transverse modules are equal. The pitch diameters of the pinion and gear are determined from the selected module and tooth numbers.

The diameter of the pinion and gear calculated with d = normal module x Number of Teeth.

$$d_p = mN_p = (3.5)(16) = 56 \; mm$$

$$d_g = mN_g = (3.5)(55) = 192.5 \; mm$$

Center distance:

$$C = \frac{d_p + d_g}{2} = \frac{56 + 192.5}{2} = 124.25 \; mm$$

For spur gears, Shigley recommends the following face width range:

$$3\pi m < b < 5\pi m$$

$$3\pi(3.5) = 32.99 < b < 5\pi(3.5) = 54.98$$

So b = 51 mm is selected.

2.4 Gear Forces

The Stage A gear forces are calculated using the following equations.

$$Tangential \; force = F_t = \frac{2T}{d_g} = \frac{2(101.77 \; Nm)}{192.5 \; mm} = 1057.35 \; N$$

$$Radial \; Force = F_r = F_t * \tan(\varphi) = (1057.35)(tan20) = 384.84 \; N$$

$$Axial \; Force = F_a = 0$$

2.5 AGMA Factors

2.5.1 Pitch-Line Velocity

$$v = \frac{\pi d_g n_{in}}{60} = \frac{\pi(0.1925)(350)}{60} = 3.53 \frac{m}{s}$$

2.5.2 Dynamic Factor

$$B = 0.25 \, (12 - Q_V)^{\frac{2}{3}} = 0.25 \, (12 - 7)^{\frac{2}{3}} = 0.731$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.731) = 65.06$$

So dynamic factor can calculated:

$$\left(\frac{A + \sqrt{200V}}{A}\right)^{B} = \left(\frac{65.06 + \sqrt{200(3.53)}}{65.06}\right)^{0.731} = 1.284$$

2.5.3 Life Factors

To calculate life factors number of cycle should determined. Number of cycles is not same for each gear because rotational speeds are not equal. Life expectance is 80000 hours is given. For pinion, rotational speed is rotational speed of intermediate shaft.

$$N_p = 60L_h n_p = 60(80000)(1203.13) = 5.78\,x\,10^9\ cycles$$

$$N_g = 60L_h n_g = 60(80000)(350) = 1.68\,x\,10^9\ cycles$$

Both design lives more than ten million cycles. According the Figure 14-14 and 14-15, line which is the below, can used because it gives a lower life factor and safer design.

$$Y_N = 1.6831(N)^{-0.0323}\ Z_N = 2.466(N)^{-0.056}$$

For pinion;

$$Y_N = 1.6831(5.78\,x\,10^9)^{-0.0323} = 0.81$$

$$Z_N = 2.466(5.78\,x\,10^9)^{-0.056} = 0.70$$

For gear;

$$Y_N = 1.6831(1.68\,x\,10^9)^{-0.0323} = 0.85$$

$$Z_N = 2.466(1.68\,x\,10^9)^{-0.056} = 0.75$$

2.5.4 Geometry Factors Bending

According the figure 14-6;

$$Y_{J,p} = 0.27\ Y_{J,g} = 0.40$$

2.5.5 Geometry Factor Wear

For a spur gear pair, the load-sharing ratio is taken as $m_N = 1$

$$The\ gear\ ratio = m_G = \frac{N_g}{N_p} = \frac{55}{16}\ \ 3.44$$

The wear geometry factor is calculated as:

$$Z_I = \frac{cos\varphi sin\varphi}{2m_N}\left(\frac{m_G}{m_G+1}\right) = \frac{(cos20)(sin20)}{2(1)}\left(\frac{3.44}{3.44+1}\right) = 0.125$$

2.5.6 Size Factor

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. So $K_s = 1$.

2.5.7 Hardness Ratio Factor

For pinion $Z_W = 1$.

$$\frac{H_{B,p}}{H_{B,g}} = \frac{350}{350} = 1 < 1.2\ So\ for\ gear\ Z_W = 1$$

2.5.8 Allowable Stresses

Spur gears are through-hardened, Grade 2 with $H_B = 350$

$$Bending: S_t = 0.703H_B + 113 = 0.703(350) + 113 = 359.1\ MPa$$

$$Wear: S_c = 2.41H_B + 237 = \ 2.41(350) + 237 = 1080.5\ MPa$$

$$\sigma_{all} = \ S_t\frac{Y_N}{Y_\theta Y_Z} \qquad\qquad \sigma_{c,all} = \ S_c\frac{Z_N}{Y_\theta Y_Z}$$

Pinion:

$$\sigma_{all} = \ S_t\frac{Y_N}{Y_\theta Y_Z} = 359.1\left(\frac{0.81}{(1)(1)}\right) = 292.4\ MPa$$

$$\sigma_{c,all} = \ S_c\frac{Z_N}{Y_\theta Y_Z} = 1080.5\left(\frac{0.7}{(1)(1)}\right) = 756.8\ MPa$$

Gear:

$$\sigma_{all} = \ S_t\frac{Y_N}{Y_\theta Y_Z} = 359.1\left(\frac{0.85}{(1)(1)}\right) = 304.3\ MPa$$

$$\sigma_{c,all} = \ S_c\frac{Z_N}{Y_\theta Y_Z} = 1080.5\left(\frac{0.75}{(1)(1)}\right) = 811\ MPa$$

2.5.9 Bending Stress Calculation

Bending stress can calculated with;

$$\sigma_p = \frac{F_t K_o K_v K_s K_H K_B}{b m_t Y_{j,p}} = \frac{(1057.3)(1.25)(1.284)(1)(1)(1)}{(51)(3.5)(0.27)} = 35.22\ MPa$$

$$\sigma_{gear} = \frac{F_t K_o K_v K_s K_H K_B}{b m_t Y_{j,g}} = \frac{(1057.3)(1.25)(1.284)(1)(1)(1)}{(51)(3.5)(0.40)} = 23.77\ MPa$$

2.5.10 Contact Stress Calculation

Contact stress can calculated with;

$$\sigma_{c,p} = \sigma_{c,g} = Z_E \sqrt{\frac{F_t K_o K_v K_s K_H Z_R}{d_p b Z_I}} = 191 \sqrt{\frac{(1057.3)(1.25)(1.284)(1)(1)(1)}{(56)(51)(0.125)}}$$

$$= 417.4\ MPa$$

2.5.11 Safety Factors

Table 3. Stage A Bending and Contact Safety Factors

	Bending, S_F	Contact, S_H
Pinion	$\frac{\sigma_{all}}{\sigma_p} = \frac{292.4}{35.22} = 8.30$	$\frac{\sigma_{c,all}}{\sigma_{c,p}} = \frac{756.8}{417.4} = 1.813$
Gear	$\frac{\sigma_{all}}{\sigma_g} = \frac{304.3}{23.77} = 12.80$	$\frac{\sigma_{c,all}}{\sigma_{c,g}} = \frac{811}{417.4} = 1.943$

The minimum safety factor for Stage A was obtained for the pinion under contact stress.

Overall FoS = 1.813

Therefore, the pinion is the critical component, and the most likely failure mode is pitting. The calculated value satisfies the target safety range of 1.8–2.0.

3 STAGE B DESIGN

Stage B is the selectable-ratio stage of the gearbox and transmits power from the intermediate shaft to the output shaft. It consists of three separate spur gear–pinion pairs, designated as V1, V2, and V3, each corresponding to a different rotor operating condition. Depending on the rotor speed, the appropriate gear pair is selected to provide the required additional speed increase and bring the generator speed close to 1500 rpm. Since all three gear pairs operate between the same intermediate and output shafts, they must have the same center distance. The design of each Stage B gear pair includes the determination of the transmission ratio, tooth numbers, module, pitch diameters, face width, gear forces, AGMA factors, bending stress, contact stress, and safety factors.

3.1 V1 Low Operating Condition

Table 4. Stage B–V1 Design Inputs and AGMA Parameters

Input Power	33.6 W
Input Rotational Speed	319.7 rpm
Driving Gear Tooth Number	104T
Driven Pinion Tooth Number	22T
Standard Full Depth	k = 1
Pressure Angle	20 degree
Helix Angle	0 degree
Selected Standard Module	1.25 mm
Selected Face Width	12 mm
Cold-Drawn AISI 1020	HB = 150
The Required Service Life	80000 hours
Overload Factor	$K_o = 1.25$
Dynamic Factor	Calculated in Report
Size Factor	$K_s = 1$
Hardness Ratio Factor	$Z_W = 1$
Load Distribution Factor	$K_H = 1$
Rim Thickness Factor	$K_B = 1$
Temperature Factor	$Y_\theta = 1$
Reliability Factor	$Y_Z = 1$
The Elastic Coefficient	$Z_E = 191\sqrt{MPa}$
Quality Number	$Q_v = 7$
Surface Condition Factor	$Z_R = 1$

3.1.1 Input Torque and Transmission Ratio

For the V1 operating condition, the rotor speed and power obtained from the Energy Systems group are 93 rpm and 33.6 W. The input torque of Stage B–V1 is calculated from the power and the intermediate-shaft speed:

$$n_{int} = 93\left(\frac{55}{16}\right) = 319.69 \text{ rpm}$$

$$T_{V1} = \frac{33.6}{2\pi(319.69)/60} = 1.004 \text{ Nm}$$

The selected V1 tooth numbers are $N_g = 104$ and $N_p = 22$.

$$i_{V1} = \frac{N_g}{N_p} = \frac{104}{22} = 4.727$$

$$n_{out} = n_{int}\, i_{V1} = 319.69(4.727) = 1511.3 \text{ rpm}$$

The target generator speed is 1500 rpm with an allowable tolerance of ±1%. Therefore, the acceptable output-speed range is 1485–1515 rpm. For V1, the calculated output speed is 1511.3 rpm. Since this value lies within the allowable range, the V1 transmission ratio satisfies the generator-speed requirement.

3.1.2 Undercut Control

For a standard full-depth spur gear, the minimum allowable pinion tooth number is calculated by:

$$N_{p,\min} = \frac{2k}{(1+2i)\sin^2\phi}\left[i+\sqrt{i^2+(1+2i)\sin^2\phi}\right]$$

Substituting the V1 values:

$$N_{p,\min} = \frac{2(1)}{[1+2(4.7273)]\sin^2 20^\circ}\left[4.7273+\sqrt{(4.7273)^2+[1+2(4.7273)]\sin^2 20^\circ}\right]$$

$$N_{p,\min} = 15.67$$

The selected V1 pinion has:

$$N_p = 22$$

Since:

$$22 > 15.67$$

the selected V1 pinion satisfies the minimum tooth-number requirement, and undercutting is avoided.

3.1.3 Gear Geometry and Face Width Selection

The V1 gear pair uses standard full-depth spur gears. The selected module and tooth numbers are:

$$m = 1.25 \text{ mm}$$

$$N_p = 22$$

$$N_g = 104$$

For spur gears, the pitch diameter is calculated by:

$$d = mN$$

Therefore, the pinion pitch diameter is:

$$d_p = mN_p = (1.25)(22) = 27.50 \text{ mm}$$

The gear pitch diameter is:

$$d_g = mN_g = (1.25)(104) = 130.00 \text{ mm}$$

The center distance of the V1 gear pair is:

$$C = \frac{d_p + d_g}{2} = \frac{27.50 + 130.00}{2} = 78.75 \text{ mm}$$

For spur gears, the recommended face-width range is:

$$3\pi m < b < 5\pi m$$

Substituting the selected module:

$$3\pi(1.25) < b < 5\pi(1.25) = 11.78 \text{ mm} < b < 19.63 \text{ mm}$$

The selected face width is: $b = 12$ mm Since

$$11.78 < 12 < 19.63$$

the selected face width satisfies the recommended range.

3.1.4 Gear Forces

The V1 gear forces are calculated using the torque transmitted through the intermediate shaft.

The tangential force is calculated as:

$$F_t = \frac{2T}{d_g} = \frac{2(1.004 \times 10^3)}{130} = 15.45\ \text{N}$$

The radial force is:

$$F_r = F_t \tan\phi = (15.45)\tan 20^\circ = 5.62\ \text{N}$$

Since spur gears do not produce axial force:

$$F_a = 0$$

3.1.5 AGMA Factors

3.1.5.1 Pitch-Line Velocity

The pitch-line velocity is calculated using the pitch diameter and rotational speed of the driving gear on the intermediate shaft.

$$v = \frac{\pi d_g n_{\text{in}}}{60} = \frac{\pi(0.130)(319.69)}{60} = 2.18\ \text{m/s}$$

3.1.5.2 Dynamic Factor

For a gear quality number of

$$Q_v = 7$$

the constants B and A are calculated as:

$$B = 0.25(12 - Q_v)^{\frac{2}{3}} = 0.25(12 - 7)^{\frac{2}{3}} = 0.731$$
$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.731) = 65.06$$

The dynamic factor is:

$$K_v = \left(\frac{A + \sqrt{200v}}{A}\right)^B = \left(\frac{65.06 + \sqrt{200(2.18)}}{65.06}\right)^{0.731} = 1.225$$

3.1.5.3 Life Factors

The number of load cycles is calculated separately for the pinion and the gear because their rotational speeds are different. The required service life is:

$$L_h = 80{,}000\ \text{h}$$

The V1 gear speed is the intermediate-shaft speed:

$$n_g = 319.69 \text{ rpm}$$

The V1 pinion speed is the output-shaft speed:

$$n_p = 1511.25 \text{ rpm}$$

For the pinion:

$$N_p = 60L_h n_p = 60(80{,}000)(1511.25) = 7.254 \times 10^9 \text{ cycles}$$

For the gear:

$$N_g = 60L_h n_g = 60(80{,}000)(319.69) = 1.5345 \times 10^9 \text{ cycles}$$

Both design lives exceed 10^7 cycles. Therefore, the lower curves in Figures 14-14 and 14-15 are used to obtain conservative life factors.

$$Y_N = 1.6831 N^{-0.0323}$$
$$Z_N = 2.466 N^{-0.056}$$

For the pinion:

$$Y_{N,p} = 1.6831(7.254 \times 10^9)^{-0.0323} = 0.808$$
$$Z_{N,p} = 2.466(7.254 \times 10^9)^{-0.056} = 0.692$$

For the gear:

$$Y_{N,g} = 1.6831(1.5345 \times 10^9)^{-0.0323} = 0.850$$
$$Z_{N,g} = 2.466(1.5345 \times 10^9)^{-0.056} = 0.754$$

3.1.5.4 Geometry Factors Bending

According to Figure 14-6, the bending geometry factors are approximately:

$$Y_{J,p} = 0.31$$
$$Y_{J,g} = 0.45$$

3.1.5.5 Geometry Factor Wear

For a spur gear pair, the load-sharing ratio is:

$$m_N = 1$$

The gear ratio is:

$$m_G = \frac{N_g}{N_p} = \frac{104}{22} = 4.7273$$

The wear geometry factor is calculated as:

$$Z_I = \frac{\cos\ \phi \sin\ \phi}{2m_N}\left(\frac{m_G}{m_G + 1}\right) = \frac{\cos\ 20° \sin\ 20°}{2(1)}\left(\frac{4.7273}{4.7273 + 1}\right) = 0.133$$

3.1.5.6 Size Factor

Standard size factors have not yet been established for gear teeth where a detrimental size effect exists. Therefore:

$$K_s = 1$$

3.1.5.7 Hardness-Ratio Factor

The pinion and gear have the same hardness:

$$H_{B,p} = 150$$

$$H_{B,g} = 150$$

Therefore:

$$\frac{H_{B,p}}{H_{B,g}} = \frac{150}{150} = 1 < 1.2$$

Thus, the hardness-ratio factor is $Z_W = 1$ for both the pinion and the gear.

3.1.5.8 V1 Allowable Stresses

The bending strength number is:

$$S_t = 0.533H_B + 88.3 = 0.533(150) + 88.3 = 168.3\ MPa$$

The contact strength number is:

$$S_c = 2.22H_B + 200 = 2.22(150) + 200 = 533\ MPa$$

The allowable bending and contact stresses are calculated using:

$$\sigma_{all} = \frac{S_t Y_N}{Y_\theta Y_Z}$$

$$\sigma_{c,all} = \frac{S_c Z_N Z_W}{Y_\theta Y_Z}$$

Pinion:

The allowable bending stress is:

$$\sigma_{\text{all},p} = \frac{S_t Y_{N,p}}{Y_\theta Y_Z} = \frac{(168.25)(0.808)}{(1)(1)} = 135.95 \text{ MPa}$$

The allowable contact stress is:

$$\sigma_{c,\text{all},p} = \frac{S_c Z_{N,p} Z_W}{Y_\theta Y_Z} = \frac{(533)(0.692)(1)}{(1)(1)} = 368.84 \text{ MPa}$$

Gear:

The allowable bending stress is:

$$\sigma_{\text{all},g} = \frac{S_t Y_{N,g}}{Y_\theta Y_Z} = \frac{(168.25)(0.850)}{(1)(1)} = 143.01 \text{ MPa}$$

The allowable contact stress is:

$$\sigma_{c,\text{all},g} = \frac{S_c Z_{N,g} Z_W}{Y_\theta Y_Z} = \frac{(533)(0.754)(1)}{(1)(1)} = 401.88 \text{ MPa}$$

3.1.5.9 V1 Bending Stress Calculation

The AGMA bending stress is calculated using:

$$\sigma_p = \frac{F_t K_o K_v K_s K_H K_B}{b m_t Y_J} = \frac{(15.45)(1.25)(1.225)(1)(1)(1)}{(12)(1.25)(0.31)} = 5.09 \text{ MPa}$$

$$\sigma_g = \frac{F_t K_o K_v K_s K_H K_B}{b m_t Y_J} = \frac{(15.45)(1.25)(1.225)(1)(1)(1)}{(12)(1.25)(0.45)} = 3.50 \text{ MPa}$$

3.1.5.10 V1 Contact Stress Calculation

The AGMA contact stress is calculated using:

$$\sigma_{c,p} = \sigma_{c,g} = Z_E \sqrt{\frac{F_t K_o K_v K_s K_H Z_R}{d_p b Z_I}} = 191 \sqrt{\frac{(15.45)(1.25)(1.225)(1)(1)(1)}{(27.5)(12)(0.133)}} = 140.23 \text{ MPa}$$

3.1.5.11 V1 Safety Factors

Table 5. Stage B–V1 Bending and Contact Safety Factors

	Bending, S_F	Contact, S_H
Pinion	$\frac{\sigma_{all}}{\sigma_p} = \frac{135.95}{5.09} = 26.71$	$\frac{\sigma_{c,all}}{\sigma_{c,p}} = \frac{368.84}{140.23} = 2.63$

Gear	$\frac{\sigma_{all}}{\sigma_g} = \frac{143.01}{3.50} = 40.86$	$\frac{\sigma_{c,all}}{\sigma_{c,g}} = \frac{401.88}{140.23} = 2.87$

The minimum safety factor for V1 is obtained for the pinion under contact stress:

$$\text{Overall FoS} = 2.63$$

Therefore, the pinion is the critical component, and the governing failure mode is contact-fatigue pitting. The V1 gear pair is safe; however, its minimum safety factor is higher than the target range of 1.8–2.0.

3.2 V2 Intermediate Operating Condition

Intermediate condition and maximum condition calculations are given in the table and shown in result tables.

Table 6. Stage B–V2 Design Calculations and Results

Parameter	Calculation	Result
Operating condition		V2 Intermediate Condition
Input power		$P = 427$ W
Rotor rotational speed		$n_{\text{rotor}} = 217$ rpm
Intermediate-shaft speed	$217\left(\frac{55}{16}\right)$	$n_{\text{in}} = 745.94$ rpm
Driving gear tooth number	—	$N_g = 84$
Driven pinion tooth number	—	$N_p = 42$
Standard full-depth factor	—	$k = 1$
Pressure angle	—	$\phi = 20°$
Helix angle	—	$\psi = 0°$
Standard module	—	$m = 1.25$ mm
Selected face width	—	$b = 12$ mm

Material	—	Grade 1 through-hardened steel
Brinell hardness	—	$H_B = 270$
Required service life	—	$L_h = 80{,}000$ h
Overload factor	—	$K_o = 1.25$
Size factor	—	$K_s = 1$
Load-distribution factor	—	$K_H = 1$
Rim-thickness factor	—	$K_B = 1$
Temperature factor	—	$Y_\theta = 1$
Reliability factor	—	$Y_Z = 1$
Hardness-ratio factor	—	$Z_W = 1$
Elastic coefficient	—	$Z_E = 191\sqrt{\text{MPa}}$
Quality number	—	$Q_v = 7$
Surface-condition factor	—	$Z_R = 1$
Input torque	$\frac{427}{2\pi(745.94)/60}$	$T_{V2} = 5.466$ Nm
Transmission ratio	$\frac{N_g}{N_p} = \frac{84}{42}$	$i_{V2} = 2.0$
Output speed	$745.94(2.000)$	$n_{\text{out}} = 1491.88$ rpm
Acceptable generator-speed range	$1500 \pm 1\%$	1485–1515 rpm

Output-speed check	$1491.88 \in [1485,1515]$	Acceptable
Minimum pinion tooth number	$\frac{2(1)}{[1+2(2)]\sin^2 20^\circ}\left[2+\sqrt{2^2+[1+2(2)]\sin^2 20^\circ}\right]$	$N_{p,\min} = 14.16$
Undercut check	$42 > 14.16$	Undercut avoided
Pinion pitch diameter	$d_p = mN_p = (1.25)(42)$	$d_p = 52.50$ mm
Gear pitch diameter	$d_g = mN_g = (1.25)(84)$	$d_g = 105.00$ mm
Center distance	$\frac{52.50+105.00}{2}$	$C = 78.75$ mm
Recommended face-width range	$3\pi(1.25) < b < 5\pi(1.25)$	$11.78 < b < 19.63$ mm
Face-width check	$11.78 < 12 < 19.63$	Acceptable
Tangential force	$\frac{2(5.466\times 10^3)}{105}$	$F_t = 104.12$ N
Radial force	$104.12\tan 20^\circ$	$F_r = 37.90$ N
Axial force	Spur gear	$F_a = 0$
Pitch-line velocity	$\frac{\pi(0.105)(745.94)}{60}$	$v = 4.10$ m/s
Dynamic-factor constant B	$0.25(12-7)^{2/3}$	$B = 0.731$
Dynamic-factor constant A	$50+56(1-0.731)$	$A = 65.06$
Dynamic factor	$\left(\frac{65.06+\sqrt{200(4.10)}}{65.06}\right)^{0.731}$	$K_v = 1.306$
Pinion rotational speed	—	$n_p = 1491.88$ rpm
Gear rotational speed	—	$n_g = 745.94$ rpm

Pinion number of cycles	$60(80{,}000)(1491.88)$	$N_p = 7.161 \times 10^9$
Gear number of cycles	$60(80{,}000)(745.94)$	$N_g = 3.5805 \times 10^9$
Pinion bending life factor	$1.6831(7.161 \times 10^9)^{-0.0323}$	$Y_{N,p} = 0.809$
Pinion contact life factor	$2.466(7.161 \times 10^9)^{-0.056}$	$Z_{N,p} = 0.692$
Gear bending life factor	$1.6831(3.5805 \times 10^9)^{-0.0323}$	$Y_{N,g} = 0.827$
Gear contact life factor	$2.466(3.5805 \times 10^9)^{-0.056}$	$Z_{N,g} = 0.719$
Pinion bending geometry factor	Figure 14-6	$Y_{J,p} = 0.38$
Gear bending geometry factor	Figure 14-6	$Y_{J,g} = 0.44$
Load-sharing ratio	Spur gear pair	$m_N = 1$
Gear ratio for wear calculation	$\frac{84}{42}$	$m_G = 2.0$
Wear geometry factor	$\frac{\cos 20° \sin 20°}{2(1)}\left(\frac{2}{2+1}\right)$	$Z_I = 0.107$
Hardness ratio	$\frac{270}{270}$	$1 < 1.2$
Hardness-ratio-factor check	Equal hardnesses	$Z_W = 1$
Bending strength number	$S_t = 0.533H_B + 88.3$	—
Bending strength number	$0.533(270) + 88.3$	$S_t = 232.21$ MPa

Contact strength number	$S_c = 2.22H_B + 200$	—
Contact strength number	$2.22(270) + 200$	S_c $= 799.40$ MPa
Pinion allowable bending stress	$\frac{(232.21)(0.809)}{(1)(1)}$	$\sigma_{\text{all},p}$ $= 187.79$ MPa
Pinion allowable contact stress	$\frac{(799.40)(0.692)(1)}{(1)(1)}$	$\sigma_{c,\text{all},p}$ $= 553.20$ MPa
Gear allowable bending stress	$\frac{(232.21)(0.827)}{(1)(1)}$	$\sigma_{\text{all},g}$ $= 192.04$ MPa
Gear allowable contact stress	$\frac{(799.40)(0.719)(1)}{(1)(1)}$	$\sigma_{c,\text{all},g}$ $= 575.09$ MPa
Pinion bending stress	$\frac{(104.12)(1.25)(1.306)(1)(1)(1)}{(12)(1.25)(0.38)}$	$\sigma_p = 29.81$ MPa
Gear bending stress	$\frac{(104.12)(1.25)(1.306)(1)(1)(1)}{(12)(1.25)(0.44)}$	$\sigma_g = 25.75$ MPa
Pinion and gear contact stress	$191\sqrt{\frac{(104.12)(1.25)(1.306)(1)(1)(1)}{(52.5)(12)(0.107)}}$	$\sigma_{c,p} = \sigma_{c,g}$ $= 303.06$ MPa
Pinion bending safety factor	$\frac{187.79}{29.81}$	$S_{F,p} = 6.30$
Pinion contact safety factor	$\frac{553.20}{303.06}$	$S_{H,p} = 1.83$
Gear bending safety factor	$\frac{192.04}{25.75}$	$S_{F,g} = 7.46$
Gear contact safety factor	$\frac{575.09}{303.06}$	$S_{H,g} = 1.90$
Overall factor of safety	Minimum of all calculated safety factors	Overall FoS $= 1.83$

Critical component	Minimum safety factor occurs under pinion contact stress	Pinion
Final evaluation	$1.8 < 1.83 < 2.0$	Design satisfies the target range

The minimum safety factor for V2 is obtained for the pinion under contact stress:

$$\text{Overall FoS} = 1.83$$

Therefore, the pinion is the critical component, and the governing failure mode is contact-fatigue pitting. The calculated value lies within the target safety-factor range of 1.8–2.0; therefore, the V2 gear-pair design satisfies the required safety criterion.

3.3 V3 Maximum Operating Condition

Table 7. Stage B–V3 Design Calculations and Results

Parameter	Calculation	Result
Operating condition	—	V3 Maximum Condition
Input power	—	$P = 3730$ W
Rotor rotational speed	—	$n_{\text{rotor}} = 350$ rpm
Intermediate -shaft speed	$350\left(\frac{55}{16}\right)$	$n_{\text{in}} = 1203.13$ rpm
Driving gear tooth number	—	$N_g = 35$
Driven pinion tooth number	—	$N_p = 28$
Standard full-depth factor	—	$k = 1$
Pressure angle	—	$\phi = 20°$

Helix angle	—	$\psi = 0°$
Standard module	—	$m = 2.50$ mm
Selected face width	—	$b = 39$ mm
Material	—	Grade 2 through-hardened steel
Brinell hardness	—	$H_B = 350$
Required service life	—	$L_h = 80{,}000$ h
Overload factor	—	$K_o = 1.25$
Size factor	—	$K_s = 1$
Load-distribution factor	—	$K_H = 1$
Rim-thickness factor	—	$K_B = 1$
Temperature factor	—	$Y_\theta = 1$
Reliability factor	—	$Y_Z = 1$
Hardness-ratio factor	—	$Z_W = 1$
Elastic coefficient	—	$Z_E = 191\sqrt{\text{MPa}}$
Quality number	—	$Q_v = 7$

Surface-condition factor	—	$Z_R = 1$
Input torque	$\frac{3730}{2\pi(1203.13)/60}$	$T_{V3} = 29.61Nm$
Transmission ratio	$\frac{N_g}{N_p} = \frac{35}{28}$	$i_{V3} = 1.250$
Output speed	$1203.13(1.250)$	$n_{\text{out}} = 1503.91$ rpm
Acceptable generator-speed range	$1500 \pm 1\%$	1485–1515 rpm
Output-speed check	$1503.91 \in [1485,1515]$	Acceptable
Minimum pinion tooth number	$\frac{2(1)}{[1+2(1.25)]\sin^2 20°}[1.25+\sqrt{1.25^2+[1+2(1.25)]\sin$	$N_{p,\min} = 12.97$
Undercut check	$28 > 12.97$	Undercut avoided
Pinion pitch diameter	$d_p = mN_p = (2.50)(28)$	$d_p = 70.00$ mm
Gear pitch diameter	$d_g = mN_g = (2.50)(35)$	$d_g = 87.50$ mm
Center distance	$\frac{70.00+87.50}{2}$	$C = 78.75$ mm
Recommended face-width range	$3\pi(2.50) < b < 5\pi(2.50)$	$23.56 < b < 39.27$ mm
Face-width check	$23.56 < 39 < 39.27$	Acceptable
Tangential force	$\frac{2(29.605 \times 10^3)}{87.5}$	$F_t = 676.69$ N

Radial force	$676.69 \tan\ 20°$	$F_r = 246.30\ \text{N}$
Axial force	Spur gear	$F_a = 0$
Pitch-line velocity	$\frac{\pi(0.0875)(1203.13)}{60}$	$v = 5.51\ \text{m/s}$
Dynamic-factor constant *B*	$0.25(12-7)^{2/3}$	$B = 0.731$
Dynamic-factor constant *A*	$50 + 56(1 - 0.731)$	$A = 65.06$
Dynamic factor	$\left(\frac{65.06 + \sqrt{200(5.51)}}{65.06}\right)^{0.731}$	$K_v = 1.352$
Pinion rotational speed	—	$n_p = 1503.91\ \text{rpm}$
Gear rotational speed	—	$n_g = 1203.13\ \text{rpm}$
Pinion number of cycles	$60(80{,}000)(1503.91)$	$N_p = 7.219 \times 10^9$
Gear number of cycles	$60(80{,}000)(1203.13)$	$N_g = 5.775 \times 10^9$
Pinion bending life factor	$1.6831(7.219 \times 10^9)^{-0.0323}$	$Y_{N,p} = 0.809$
Pinion contact life factor	$2.466(7.219 \times 10^9)^{-0.056}$	$Z_{N,p} = 0.692$

Gear bending life factor	$1.6831(5.775 \times 10^9)^{-0.0323}$	$Y_{N,g} = 0.814$
Gear contact life factor	$2.466(5.775 \times 10^9)^{-0.056}$	$Z_{N,g} = 0.700$
Pinion bending geometry factor	Figure 14-6	$Y_{J,p} = 0.34$
Gear bending geometry factor	Figure 14-6	$Y_{J,g} = 0.36$
Load-sharing ratio	Spur gear pair	$m_N = 1$
Gear ratio for wear calculation	$\frac{35}{28}$	$m_G = 1.250$
Wear geometry factor	$\frac{\cos 20° \sin 20°}{2(1)}\left(\frac{1.25}{1.25+1}\right)$	$Z_I = 0.0893$
Hardness ratio	$\frac{350}{350}$	$1 < 1.2$
Hardness-ratio-factor check	Equal hardnesses	$Z_W = 1$
Bending strength number	$S_t = 0.703H_B + 113$	—
Bending strength number	$0.703(350) + 113$	$S_t = 359.05$ MPa

Contact strength number	$S_c = 2.41H_B + 237$	—
Contact strength number	$2.41(350) + 237$	$S_c = 1080.50$ MPa
Pinion allowable bending stress	$\frac{(359.05)(0.809)}{(1)(1)}$	$\sigma_{\text{all},p} = 290.29$ MPa
Pinion allowable contact stress	$\frac{(1080.50)(0.692)(1)}{(1)(1)}$	$\sigma_{c,\text{all},p} = 747.38$ MPa
Gear allowable bending stress	$\frac{(359.05)(0.814)}{(1)(1)}$	$\sigma_{\text{all},g} = 292.39$ MPa
Gear allowable contact stress	$\frac{(1080.50)(0.700)(1)}{(1)(1)}$	$\sigma_{c,\text{all},g} = 756.78$ MPa
Pinion bending stress	$\frac{(676.69)(1.25)(1.352)(1)(1)(1)}{(39)(2.50)(0.34)}$	$\sigma_p = 34.49$ MPa
Gear bending stress	$\frac{(676.69)(1.25)(1.352)(1)(1)(1)}{(39)(2.50)(0.36)}$	$\sigma_g = 32.58$ MPa
Pinion and gear contact stress	$191\sqrt{\frac{(676.69)(1.25)(1.352)(1)(1)(1)}{(70)(39)(0.0893)}}$	$\sigma_{c,p} = \sigma_{c,g} = 413.70$ MPa

Pinion bending safety factor	$\frac{290.29}{34.49}$	$S_{F,p} = 8.42$
Pinion contact safety factor	$\frac{747.38}{413.70}$	$S_{H,p} = 1.81$
Gear bending safety factor	$\frac{292.39}{32.58}$	$S_{F,g} = 8.98$
Gear contact safety factor	$\frac{756.78}{413.70}$	$S_{H,g} = 1.83$
Overall factor of safety	Minimum of all calculated safety factors	Overall FoS = 1.81
Critical component	Minimum safety factor occurs under pinion contact stress	Pinion
Final evaluation	$1.8 < 1.81 < 2.0$	Design satisfies the target range

The minimum safety factor for V3 is obtained for the pinion under contact stress:

$$\text{Overall FoS} = 1.81$$

Therefore, the pinion is the critical component, and the governing failure mode is contact-fatigue pitting. The calculated value lies within the target safety-factor range of 1.8–2.0; therefore, the V3 gear-pair design satisfies the required safety criterion.

4 SUMMARY OF RESULTS

4.1 Stage A Results

Table 8. Summary of Stage A Design Results

Result	**Value**
Input power	3730 W

Input rotational speed	350 rpm
Driving gear / driven pinion	55/16
Module	3.5 mm
Face width	51 mm
Material	Grade 2 through-hardened steel
Brinell hardness	$H_B = 350$
Gear pitch diameter	$d_g = 192.5$ mm
Pinion pitch diameter	$d_p = 56.0$ mm
Center distance	124.25 mm
Intermediate-shaft speed	1203.13 rpm
Tangential force	$F_t = 1057.35$ N
Radial force	$F_r = 384.84$ N
Axial force	$F_a = 0$
Dynamic factor	$K_v = 1.284$
Pinion bending geometry factor	$Y_{J,p} = 0.27$
Gear bending geometry factor	$Y_{J,g} = 0.40$
Wear geometry factor	$Z_I = 0.125$
Pinion allowable bending stress	292.4 MPa
Gear allowable bending stress	304.3 MPa
Pinion allowable contact stress	756.8 MPa
Gear allowable contact stress	811.0 MPa
Pinion bending stress	35.22 MPa
Gear bending stress	23.77 MPa

Contact stress	417.4 MPa
Pinion bending safety factor	$S_{F,p} = 8.30$
Gear bending safety factor	$S_{F,g} = 12.80$
Pinion contact safety factor	$S_{H,p} = 1.813$
Gear contact safety factor	$S_{H,g} = 1.943$
Overall factor of safety	1.813
Critical component	Pinion

4.2 Stage B V1 Results

Table 9. Summary of Stage B–V1 Design Results

Result	**Value**
Operating condition	Low condition
Input power	33.6 W
Input rotational speed	319.69 rpm
Driving gear / driven pinion	104/22
Module	1.25 mm
Face width	12 mm
Material	Cold-Drawn AISI 1020
Brinell hardness	$H_B = 150$
Transmission ratio	4.7273
Output speed	1511.25 rpm
Pinion pitch diameter	$d_p = 27.50$ mm
Gear pitch diameter	$d_g = 130.00$ mm

Center distance	78.75 mm
Tangential force	$F_t = 15.45$ N
Radial force	$F_r = 5.62$ N
Axial force	$F_a = 0$
Dynamic factor	$K_v = 1.225$
Pinion bending geometry factor	$Y_{J,p} = 0.31$
Gear bending geometry factor	$Y_{J,g} = 0.45$
Wear geometry factor	$Z_I = 0.133$
Pinion allowable bending stress	135.95 MPa
Gear allowable bending stress	143.01 MPa
Pinion allowable contact stress	368.84 MPa
Gear allowable contact stress	401.88 MPa
Pinion bending stress	5.09 MPa
Gear bending stress	3.50 MPa
Contact stress	140.23 MPa
Pinion bending safety factor	$S_{F,p} = 26.71$
Gear bending safety factor	$S_{F,g} = 40.86$
Pinion contact safety factor	$S_{H,p} = 2.63$
Gear contact safety factor	$S_{H,g} = 2.87$
Overall factor of safety	2.63
Critical component	Pinion

4.3 Stage B V2 Results

Table 10. Summary of Stage B–V2 Design Results

Result	**Value**
Operating condition	Intermediate condition
Input power	427 W
Input rotational speed	745.94 rpm
Driving gear / driven pinion	84/42
Module	1.25 mm
Face width	12 mm
Material	Grade 1 through-hardened steel
Brinell hardness	$H_B = 270$
Transmission ratio	2.000
Output speed	1491.88 rpm
Pinion pitch diameter	$d_p = 52.50$ mm
Gear pitch diameter	$d_g = 105.00$ mm
Center distance	78.75 mm
Tangential force	$F_t = 104.12$ N
Radial force	$F_r = 37.90$ N
Axial force	$F_a = 0$
Dynamic factor	$K_v = 1.306$
Pinion bending geometry factor	$Y_{J,p} = 0.38$
Gear bending geometry factor	$Y_{J,g} = 0.44$
Wear geometry factor	$Z_I = 0.107$

Pinion allowable bending stress	187.79 MPa
Gear allowable bending stress	192.04 MPa
Pinion allowable contact stress	553.20 MPa
Gear allowable contact stress	575.09 MPa
Pinion bending stress	29.81 MPa
Gear bending stress	25.75 MPa
Contact stress	303.06 MPa
Pinion bending safety factor	$S_{F,p} = 6.30$
Gear bending safety factor	$S_{F,g} = 7.46$
Pinion contact safety factor	$S_{H,p} = 1.83$
Gear contact safety factor	$S_{H,g} = 1.90$
Overall factor of safety	1.83
Critical component	Pinion

4.4 Stage B V3 Results

Table 11. Summary of Stage B–V3 Design Results

Result	**Value**
Operating condition	Maximum condition
Input power	3730 W
Input rotational speed	1203.13 rpm
Driving gear / driven pinion	35/28
Module	2.50 mm
Face width	39 mm

Material	Grade 2 through-hardened steel
Brinell hardness	$H_B = 350$
Transmission ratio	1.250
Output speed	1503.91 rpm
Pinion pitch diameter	$d_p = 70.00$ mm
Gear pitch diameter	$d_g = 87.50$ mm
Center distance	78.75 mm
Tangential force	$F_t = 676.69$ N
Radial force	$F_r = 246.30$ N
Axial force	$F_a = 0$
Dynamic factor	$K_v = 1.352$
Pinion bending geometry factor	$Y_{J,p} = 0.34$
Gear bending geometry factor	$Y_{J,g} = 0.36$
Wear geometry factor	$Z_I = 0.0893$
Pinion allowable bending stress	290.29 MPa
Gear allowable bending stress	292.39 MPa
Pinion allowable contact stress	747.38 MPa
Gear allowable contact stress	756.78 MPa
Pinion bending stress	34.49 MPa
Gear bending stress	32.58 MPa
Contact stress	413.70 MPa
Pinion bending safety factor	$S_{F,p} = 8.42$
Gear bending safety factor	$S_{F,g} = 8.98$

Pinion contact safety factor	$S_{H,p} = 1.81$
Gear contact safety factor	$S_{H,g} = 1.83$
Overall factor of safety	1.81
Critical component	Pinion

Table 12. Summary of Critical Safety Factors for All Gear Pairs

Gear pair	**Critical component**	**Governing mode**	**Overall FoS**
Stage A	Pinion	Contact-fatigue pitting	1.813
Stage B–V1	Pinion	Contact-fatigue pitting	2.63
Stage B–V2	Pinion	Contact-fatigue pitting	1.83
Stage B–V3	Pinion	Contact-fatigue pitting	1.81

5 RESULTS AND DISCUSSION

The primary objective of this project was to develop a compact and economical gearbox while obtaining critical safety factors as close as practical to the target range of 1.8–2.0. The V1 gear pair remained above this range because of its low transmitted load and the common center-distance constraint. Based on the aerodynamic analyses provided by the ESE312 Energy Systems group, the inputs correspond to a small-scale, home-use wind turbine with a only 1-meter blade radius. The turbine generates mechanical power ranging from a low condition of 33.6 W (with 1.004 Nm intermediate torque) up to a maximum condition of 3730 W (with 101.8 Nm input torque).

To successfully increase these relatively low rotor speeds to the target generator speed of approximately 1500 rpm, a two-stage, three-shaft transmission with 4 spur pinions and 4 spur gears was designed. The design decisions, parameter selections, and AGMA calculations for this process are detailed below.

5.1 System Architecture and Gear Type Selection

The gearbox is divided into two parts. First part is a fixed-ratio first stage (Stage A) and second part is a selectable-ratio second stage (Stage B) for different input speeds. Spur gears were chosen over helical gears for both stages. Here are the reasons:

- Zero Axial Load: Spur gears do not produce axial forces during operation ($F_a = 0$). This eliminates the need for expensive and complex thrust bearings (e.g., tapered roller bearings), that allows for simpler shaft designs and standard bearing selections.
- Manufacturing and Assembly: Spur gears have simpler geometries, have lower manufacturing costs and easier for assembly, which is ideal for a cost-effective, home-use turbine concept.
- Acoustic Considerations: Since spur gears are known to make more noise at high speeds (such as the 1500 rpm target) compared to helical gears, acoustic limitations and vibration were not specified as design constraints in the project requirements. Therefore, the simpler and cheaper selection of Spur gears are done.

5.2 AGMA Factors and Operational Constraints

The gear design and stress evaluations were performed strictly according to AGMA procedures. Standard factors were determined based on the physical realities of the design:

- Overload Factor: An overload factor of 1.25 was selected for all operating conditions. Because the turbine is small-scale with low-inertia 1-meter blades, aerodynamic gusts will cause the rotor to rapidly accelerate rather than transmit heavy shock loads to the drivetrain, justifying a "moderate shock" assumption.
- Life Factors: The design targets a minimum service life of 80,000 hours. Because the high-speed Stage B output shaft reaches ~1500 rpm, the total number of load cycles drastically exceeds 10^7 cycles (reaching over 7×10^9 cycles). As a result, the bending and contact life factors (Y_N and Z_N) chosen for the allowable stresses. Lower-bound curves from AGMA standard figures were used to ensure conservative and safe life factors.
- Standard Factors: Size (K_s), Load Distribution (K_H), Rim Thickness (K_B), Temperature (Y_θ), Reliability (Y_Z), and Surface Condition (Z_R) factors were all evaluated as 1.0, assuming ideal manufacturing and operational environments.

5.3 Stage A: Maximum Load Configuration

Stage A connects the rotor input shaft to the intermediate shaft and operates as a fixed speed-increaser.

- Design Condition: Stage A was designed based on the maximum wind condition, handling an input power of 3730 W and the system's highest torque of 101.8 Nm.
- Material Selection: To withstand the high input torque and prevent surface pitting, a strong Grade 2 through-hardened steel with a Brinell hardness of 350 HB was selected.
- Evaluation: With a standard module of 3.5 mm and a face width of 51 mm, the Stage A gears successfully avoided undercutting. The desired safety factor region was obtained on the pinion under contact stress, yielding an overall FoS of 1.813. This shows that pitting is the primary failure mode, however the design perfectly aligns with the target FoS range of 1.8 to 2.0.

5.4 Stage B: Selectable Ratio and Center Distance Optimization

Stage B transmits power from the intermediate shaft to the output generator shaft using three selectable spur gear-pinion pairs (V1, V2, V3).

- Center Distance Constraint: The most critical mechanical constraint in Stage B was that all three gear sets must satisfy an identical center distance. By carefully adjusting standard modules and tooth numbers, the center distance for all three conditions was successfully locked at 78.75 mm.
- V1 (Low Condition) Optimization: In the V1 condition, the transmitted torque drops to approximately 1.004 Nm. Due to the strict center distance constraint of 78.75 mm, the gear geometry could not be reduced any further.So that, evaluating this large geometry against a negligible 1 Nm load naturally caused really high safety factors. To prevent over-engineering, reduce manufacturing costs and also lower the FoS to a more reasonable level, a highly economical Grade 1 unhardened steel with a hardness of 150 HB (cold-drawn AISI 1020) was selected. Despite the massive cycle count and life factor penalties, the V1 pinion maintained a highly secure contact FoS of 2.63.
- V2 and V3 Conditions: The intermediate (V2) and maximum (V3) gear pairs managed higher loads of 5.466 Nm and 29.61 Nm, respectively. By appropriately

scaling the material hardness, using 270 HB for V2 and Grade 2 350 HB for V3, the system was optimized without changing the core geometry.

5.5 Final Safety Factor Evaluation

Through material selection and geometry scaling, the main failure mode across the entire gearbox was contact-fatigue pitting on the pinions. The final overall safety factors for the critical components are below:

- Stage A: 1.813
- Stage B (V1): 2.63
- Stage B (V2): 1.83
- Stage B (V3): 1.81

Stage A, V2, and V3 successfully satisfy the target safety-factor range of 1.8–2.0. The V1 safety factor remains above this range because of the very low transmitted load and the common center-distance requirement. Further reduction of the V1 safety factor was not considered practical while maintaining the selected standard module, minimum tooth-number requirement, and manufacturable gear geometry. Overall, the resulting gearbox is compact, structurally adequate, cost-effective, and tailored to the aerodynamic operating conditions of the small-scale wind turbine.

6 CONCLUSION

In this project, we successfully designed an optimal, compact, and cost-effective gearbox. This gearbox transmits mechanical power for a home-use wind turbine with a 1-meter blade radius. We used data from the Energy Systems (ESE312) group. The aerodynamic power ranges from 33.6 W to 3730 W. The maximum rotor torque is 101.8 Nm. Our two-stage, three-shaft gearbox system increases the rotor speed to the target generator speed of ~1500 rpm.

We chose standard full-depth spur gears for the system architecture. This choice drops the axial forces to zero. It also reduces manufacturing costs and allows easy assembly for the home-use concept. Our design targets 98% reliability and a minimum service life of 80,000 hours. We performed detailed bending and contact (pitting) fatigue analyses according to AGMA standards.

The final project results from our numerical analyses and optimizations are below:

- Stage A (Fixed-Ratio Input Stage): This stage handles the maximum system torque of 101.8 Nm. We used a Grade 2 through-hardened steel with 350 HB hardness here. Despite the heavy loads, contact stress on the pinion is the critical factor. This stage perfectly reaches the target range with a Factor of Safety (FoS) of 1.813.
- Stage B (Selectable-Ratio Output Stage): This stage has 3 different gears (V1, V2, V3) on the same shaft. We successfully locked their center distance at 78.75 mm.
- Optimization and Material Management:

  - V3 (Maximum Wind Condition): We used 350 HB steel and achieved an FoS of 1.81.
  - V2 (Intermediate Wind Condition): The load decreases here. So, we reduced the material hardness to 270 HB. We reached an FoS of 1.83.
  - V1 (Low Wind Condition): The transmitted torque drops to only ~1.004 Nm. However, the center distance must stay at 78.75 mm. Therefore, we could not make the gear geometry any smaller. We wanted to prevent over-engineering and minimize costs. So, we intentionally chose a cold-drawn AISI 1020 standard industrial steel with 150 HB hardness. Despite this cheap choice, the V1 pinion provided a high and very safe FoS of 2.63.

In conclusion, contact-fatigue (pitting) on the pinions is the main failure mode for all stages in the gearbox. The V1 condition is geometrically limited because of the center distance rule. Except for V1, all critical parts in the system perfectly met the target FoS range of 1.8 to 2.0.

We designed this mechanical transmission system using Machine Elements principles. It successfully meets all project rules and AGMA safety requirements. It shows great structural integrity, speed/life optimization, and smart material choices.

7 TEAM MEETINGS

Table 13. Project Meeting Schedule and Team Participation

Date	Meeting Time	Subject	Attendees
May 29, 2026	1 hours	The team gathered for an initial brainstorming session on the 3-speed gearbox configuration and successfully retrieved the rotor aerodynamic load and target speed datasets from the ESE group.	Efe Ömer Aydın Berat Balmış Samet Yavuz Samet Aslan Batuhan Bal
June 5, 2026	2 hours	A status review was conducted, and the group established a collaborative Excel computing sheet to automate the iterative AGMA bending and contact stress equations.	Efe Ömer Aydın Berat Balmış Samet Yavuz Samet Aslan Batuhan Bal
June 12, 2026	1 hours 30 minute	The team evaluated the compatibility of the ESE turbine design parameters and decided to strictly adhere to the initial baseline inputs to ensure compact geometry and prevent an over-safe design.	Efe Ömer Aydın Berat Balmış Samet Yavuz Samet Aslan Batuhan Bal

8 APPENDIX

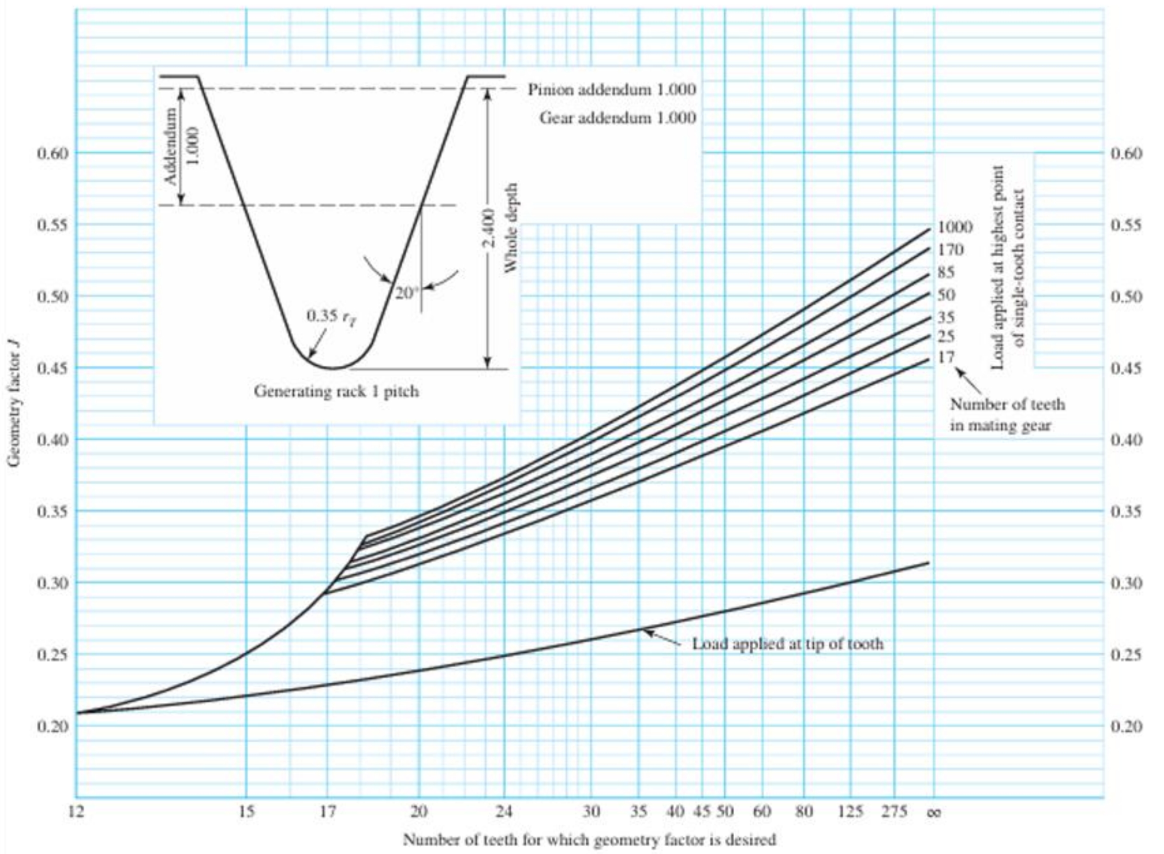

Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Figure 1. AGMA Bending Geometry Factor Yj for 20 Full-Depth Spur Gears

Figure 14–2

Gear bending strength for through-hardened steels, S_t. The SI equations are: $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

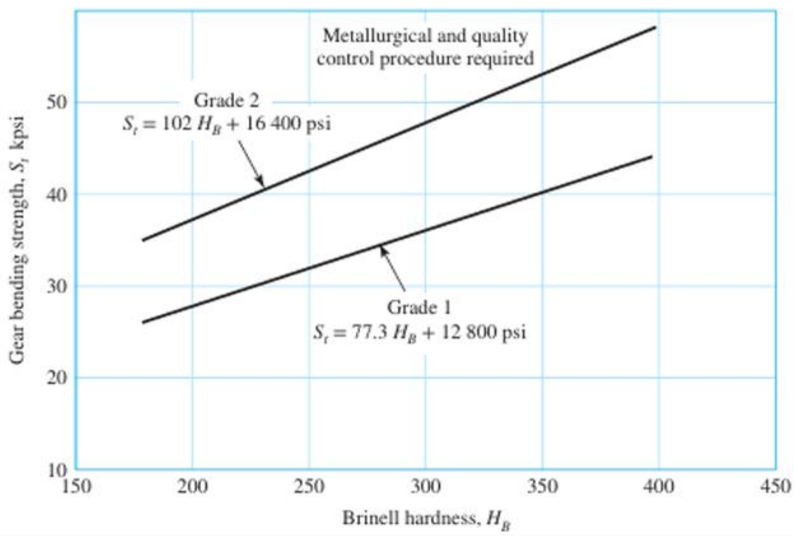

Figure 2. Allowable Bending Stress Number St for Through-Hardened Steel Gears

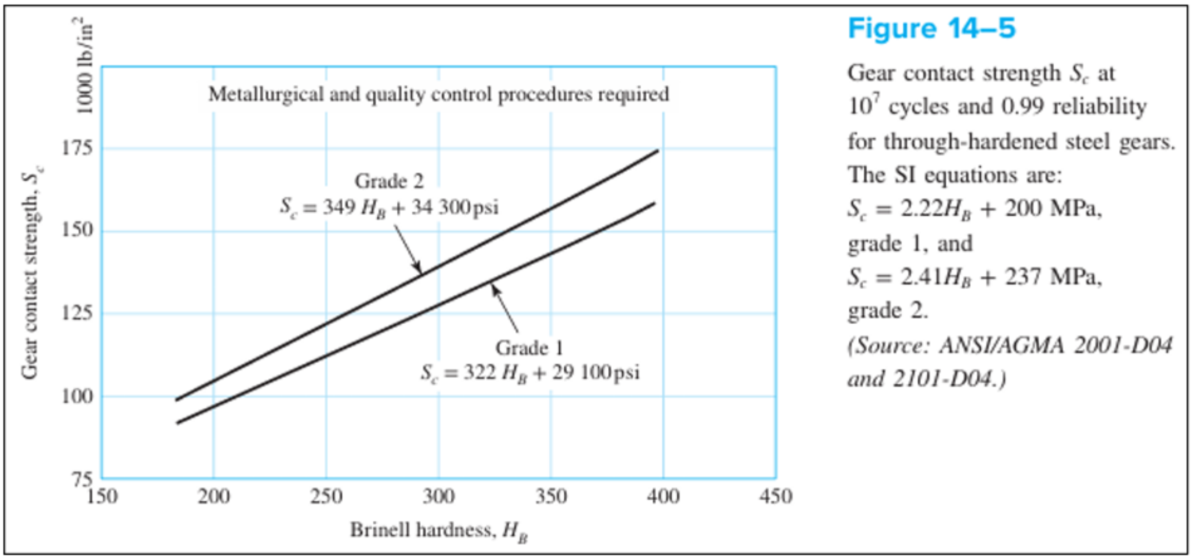

Figure 3. Allowable Contact Stress Number Sc for Through-Hardened Steel Gears

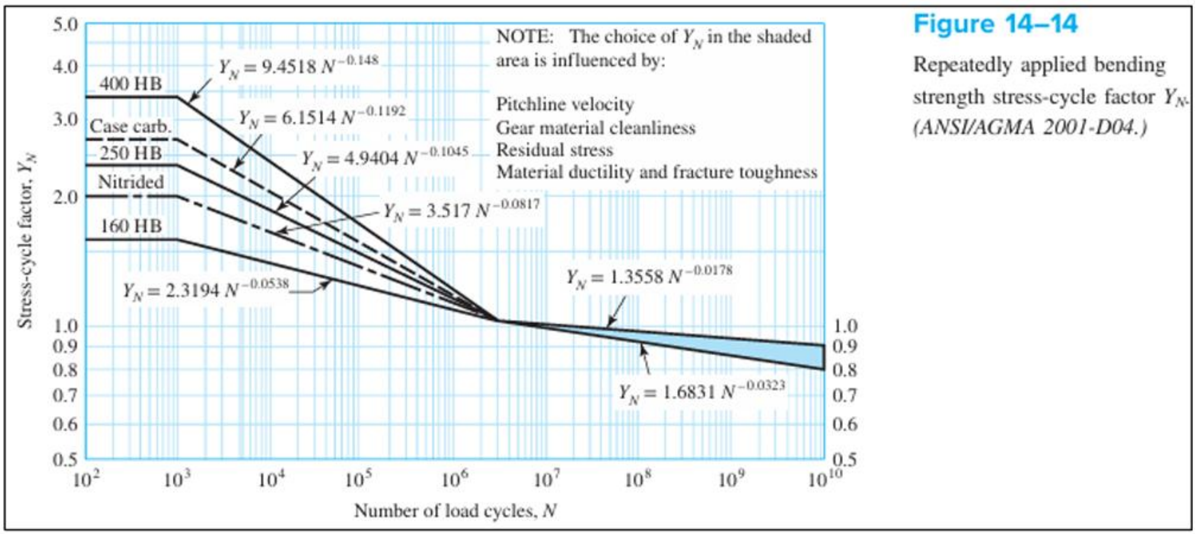

Figure 4. Bending Stress-Cycle Factor YN for Through-Hardened Spur Gears

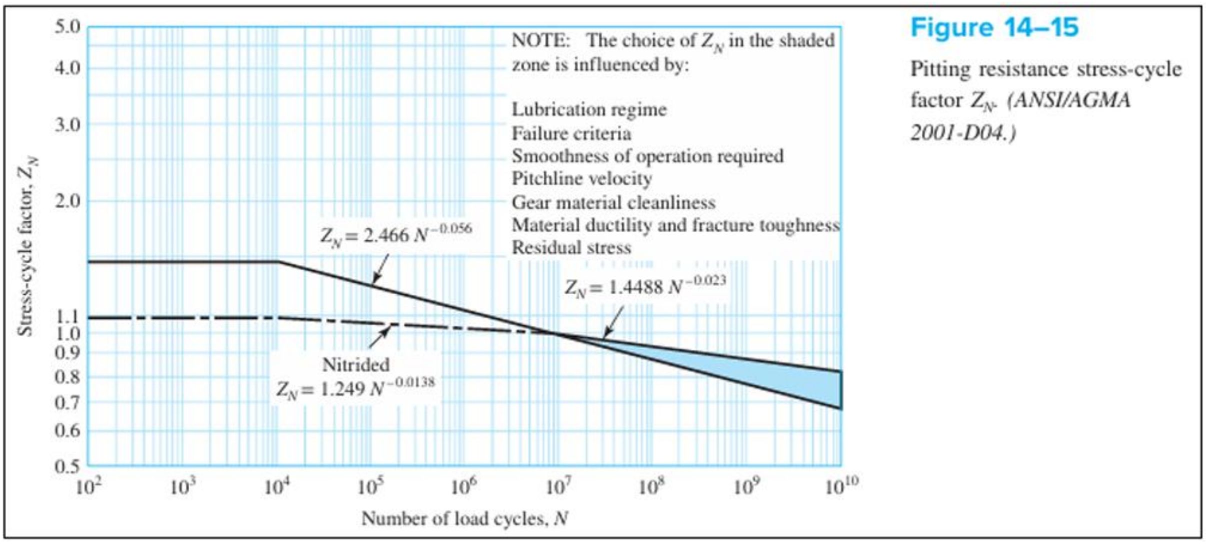

Figure 5. Contact Stress-Cycle Factor ZN for Through-Hardened Gears

9 REFERENCES

Budynas, R. G., & Nisbett, J. K. (2020). *Shigley's mechanical engineering design* (11th ed., Chapter 14: Spur and helical gears). McGraw-Hill Education.